Studies of the Optics of Neutrons. II. Spin-Independent Interaction between Neutrons and Electrons

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After discussing the three known experimental methods for determining the spin-independent neutron-electron interaction, we show the various drawbacks of each. In particular, the method based on measuring an angle of total reflection and thereby deducing under proper assumptions the value for the amplitude of the neutron-electron interaction is shown to carry with it a grave uncertainty arising from the fact that the amplitudes compared in the reflection experiments have been deduced from cross-section measurements which contain a poorly determined contribution from incoherent scattering.

I. INTRODUCTION

THE spin-independent interaction between neutrons and electrons is assumed to be largely caused by the virtual emission of a negatively charged particle (π^-) by the neutron, whereby the neutron is left in the state of a proton.[1] The new system of a centrally located proton surrounded by a negatively charged cloud has a resultant potential energy upon the electron. This so-defined potential energy is in its spin-dependent part responsible for the exact value of the neutron's and proton's magnetic moment. Theory has so far been unable to account for these magnetic moments quantitatively and so it is not very surprising if no good quantitative values are theoretically available as far as a spin-independent interaction energy between neutron and electron is concerned. At the present state of knowledge, we are forced to be satisfied with experimental determinations.

Three methods are known by which the neutron-electron energy can be, and has been, measured. These are: (1) The determination of the differential cross section of the neutron scattering at two fairly widely distant angles. (2) The determination of the integral scattering cross sections of neutrons as a function of the wavelength. (3) The measurement of nuclear amplitudes by the determination of an angle of total reflection and correction for the effect of neutron-electron interaction.

The three methods, while giving approximately the same results for the neutron-electron interaction, show a remarkable behavior of this interaction. They seem to indicate (cf. Hughes[1]) that all the interaction present comes from the interaction of the magnetic moment of the neutron with the electron, which is looked upon as a Dirac particle; no contribution seems to be made by the virtual state $\pi^-+\text{proton}=\text{neutron}$ mentioned above. Furthermore, it is generally assumed that the neutron-optical method of determining an angle of total reflection is the most accurate one. We now wish to describe these methods individually and to indicate which drawbacks seem to be attached to them.

[1] For a description of the effects discussed and for a rather complete list of references, the reader is referred to an article by D. J. Hughes [Ann. Rev. Nucl. Sci. **3**, 93 (1953)].

II. MEASUREMENT OF THE DIFFERENTIAL CROSS SECTION FOR NEUTRON SCATTERING AT VARIOUS SCATTERING ANGLES

This method was originated by Fermi and later on refined by Hamermesh and collaborators. (For references, see Hughes.[1]) Denoting by a_n the scattering amplitude of the nucleus for slow neutrons, and by a_e that of electrons, we obtain for the differential cross section of scattering the expression

$$\sigma=[a_n+Za_eF(\theta)]^2. \tag{1}$$

Here, Z denotes the atomic number of the nucleus and $F(\theta)$ is the form factor of electronic scattering which is essentially determined by the wavelength of the neutron and the electron configuration around the nucleus. The result has to be corrected for the thermal motion of the nuclei which is no longer negligible as compared to the velocity of the neutron. In fact, it could be shown that the difference in the total scattering at two different angles arising from the hypothetical neutron-electron interaction is of the same order as the correction arising from the thermal motion of the nuclei. Since the experiments were made with neutrons, the velocity of which extended over a broad band, it was necessary to assume an expression for this velocity distribution to carry out the calculation, and it was assumed that the neutrons follow a Maxwell-like law. However, in a communication published some years ago together with Chu,[2] it was shown that the main contribution from the correction comes from the distribution of neutrons over extremely long wavelengths—in other words, from a region in which Maxwell's law must be supposed not to hold any longer. Since, on the other hand, the correction is of the same order as the effect, one cannot make any exact quantitative statement about the neutron-electron interaction.

III. VARIATION OF THE INTEGRAL CROSS SECTION WITH THE NEUTRON WAVELENGTH

These investigations, carried out at Columbia University,[1] rest on the following principle: In observations of the integral cross section of scattering from a

[2] O. Halpern and L. Chu, Phys. Rev. **86**, 594 (1951).

condensed body, the result should be, in first approximation, wavelength-independent if all the scattering is s scattering from the nucleus alone. If the scattering from electrons is coherently superposed, we obtain a wavelength dependence since, as mentioned before, there will be an angle-dependent form factor arising from the neutron-electron interaction. Thus, the observation of the wavelength dependence of the total scattering cross section gives us some indication about the rather small neutron-electron effect. Unfortunately, here too, corrections from binding forces of the nuclei and from their thermal motion, etc., have to be considered. They are at least of the order of magnitude of the looked-for effect and, while some authors[1] think that they can be accounted for with great accuracy, the question is still probably open.

IV. DETERMINATION OF THE NEUTRON-ELECTRON AMPLITUDE BY MEASUREMENT OF AN ANGLE OF TOTAL REFLECTION

This procedure, which is generally considered to be the most accurate method for measuring the neutron-electron interaction, consists of various steps leading to an accurate measurement of the scattering amplitude. The first of these steps is made by comparing the integral cross sections of scattering for two substances. Assuming that the two cross sections arise only from *coherent* scattering, these two scattering cross sections are, respectively, given by

$$(a_{n_1}+Z_1a_eF(\theta)_1)^2, \quad \text{and} \quad (a_{n_2}+Z_2a_eF(\theta)_2)^2. \tag{2}$$

It is therein assumed that the small correction

$$Z_1a_eF(\theta)_1, \quad Z_2a_eF(\theta)_2$$

can be accounted for in the end. If the cross sections are thus determined, they lead to very accurate values for the nuclear amplitudes of the two substances. The relative index of refraction for the transition of a neutron beam from the first into the second substance is then given by the expression

$$n_{1,2}=\frac{1-[N_1\lambda^2(a_1+Z_1a_eF(\theta)_1)/2\pi]_i}{1-[N_2\lambda^2(a_2+Z_2a_eF(\theta)_2)/2\pi]}. \tag{3}$$

The symbols in Eq. (3) are conventional and they also already take into account the possible neutron-electron interaction. While this neutron-electron interaction gave only a small correction to the scattering cross sections, mostly on account of the small value of its form factor for most angles, we can in Eq. (3) set this form factor equal to one, since the angle of refraction (or total reflection) is very small. Measuring now the angle of total reflection at the interface of the two substances, we obtain with the aid of Eq. (3) an expression for a_e. Since this neutron-optical measurement can be made with very great accuracy, the result obtained is more reliable than that given by any other type of measurement.

Unfortunately, accurate though the experimental method may be, the theoretical assumption that the cross sections are due only to coherent scattering is not fulfilled in the case of practical interest. It seemed indicated to use, as the two substances at the interface of which total reflection should be observed, solid Bi and solid (frozen) O_2. The expressions N_1a_1 and N_2a_2 for these two substances are nearly equal, so the resulting index of refraction is very small and, therefore, any influence of the two neutron-electron amplitudes most conspicuous. But the scattering contains a not too well known but rather sizeable incoherent component. To appreciate the magnitude of these incoherent effects, one proceeds as follows: We take for the total amplitudes of Bi and O_2 the values of approximately 8.63×10^{-13} and 5.81×10^{-13} cm measured at a wavelength corresponding to about 10 eV. According to the authors (Hughes[1]), these values were obtainable only with sufficient accuracy because they constituted a relative measurement of the amplitudes; without any sufficient theoretical basis, it was furthermore—and quite wrongly—assumed that these amplitudes were amplitudes of coherent scattering. Already the very inaccurate application of the Debye-Waller formula shows the presence of a sizeable inelastic contribution. If one should attempt to avoid this inelastic contribution by making the observation at very long wavelengths, another grave error enters through the presence of paramagnetic scattering of[3] O_2. This scattering, which is vanishingly small through form-factor action at short wavelengths, becomes rather sizeable but inaccurately determined at long wavelengths. Nothing is thereby said about further complications arising from Bi. The necessary accuracy of a small fraction of 2% is obviously unattainable.

In this analysis, it was always assumed that the contribution mentioned before of the neutron-electron interaction to the integral scattering cross section of Bi and O_2 is, due to the form factor, small enough to be only a correction which can be taken care of at the end. The result of this investigation is, therefore, that the neutron-electron interaction may be (absolutely speaking) larger than the value given by Hughes and his collaborators,[1] unless for very short wavelengths the corrections for inelastic scattering for Bi are larger than for O_2.

[3] O. Halpern and G. L. Appleton, Phys. Rev. **90**, 869 (1953). We want to use this opportunity to ask the reader to delete paragraph 3 of the paper quoted. There are no numerical changes involved in the final results given in the paper.